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ABSTRACT

In mathematics, situations calling for reconstruction of a transformation are frequent. In such situations, children aged 6 to 7 concentrate on the states, combining them in different ways. The present research aims to illustrate that difficulties and strategies observed with children 6-7 years old are not specific to this age level, but are also encountered by children up to age 12 and probably older. Numerous situations of reconstruction, ranging from easy to more complex, were prepared relying on criteria defined in a reference framework elaborated in the light of past research on problem solving. These items were presented to primary school children at all levels. Results reveal that the same errors reappear constantly, bringing to light important obstacles that interfere with the construction of a very fundamental concept in mathematics and sciences, that of transformation. (Author/PK)

RECONSTRUCTION OF A TRANSFORMATION A PROBLEM IN MATHEMATICS

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ABSTRACT

Some aspect of the arithmetic operation concept cause much more difficulties than others to young children, those in which they have to mentally reconstruct a transformation. Piaget and Inhelder's experiments on mental image single out the complexity involved in the mental reconstruction of a transformation. One needs to represent the actions that occur and anticipate events. For this to happen, Piaget asserts that the child should master reversibility that characterizes the concrete operational stage.

In mathematics, situations calling for reconstruction of a transformation are frequent. In such situations, children aged 6 to 7 concentrate on the states, combining them in different ways. The present research aims to illustrate that difficulties and strategies observed with children 6-7 years old are not specific to this age level, but are also encountered by children up to age 12 and probably older. This brings us to consider the problem not as a local one but rather as an important developmental one.

Numerous situations of reconstruction, ranging from easy to more complex, were prepared relying on criteria defined in a reference framework elaborated in the light of past research on problem solving. These items were presented to primary school children at all levels.

Results reveal that the same errors reappear constantly, putting in light important obstacles that interfere with the construction of the very fundamental concept in mathematics and sciences, that of transformation.

AIMS

Situations requiring the reconstruction of a transformation in mathematics are complex ones. These consist briefly of temporal situations starting with an initial state which is transformed into a final one. The child must recognize these situations as such, prior to identifying the unchanged characteristics and to formulate the agent of change.

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Our aims are to bring more knowledge on the complexity of reconstruction tasks:

1- by showing that they cause problems at different levels of primary school (6-7 to 11-12 years old) and, then, that they are not typical of a specific level. We are then forced to consider the problem as an important one.

2- by identifying the major difficulties they arouse and by pointing out the errors persisting during the primary school years.

METHOD & SUBJECTS

• Reference framework: A reference framework is used to construct a set of problems requiring the reconstruction of a transformation, ranked in terms of complexity. This work is based on a conceptual analysis taking into account the following variables:

Problem structure: the position of the unkown in a problem involving a unique change or a sequence of changes (opposite or not).
Nature of states: collections, magnitudes, positions.
Type of numbers: Discrete or continuous.
Number size.
Nature of the operated change: transformation or displacement.

• Population: The children are from a middle class primary school of the Montreal Catholic school board. Two classes are retained at each level from first grade (6-7 years old) to sixth grade (11-12 years old).

• The experiment:

- A selection of a set of ten complex but accessible problems is presented for each level. A pre experiment is conducted that supplies preliminary results for the different problems.
- Booklets are constitued of ten pages, each containing a problem; problems are randomly ordered in the booklet.
- The first six problems only are to be solved in order to assure fair conditions of experimentation.
- Each page of the booklets is coded with three numbers: identification of the child, grade level, order of the problem in the random sequence of ten. These procedures are used to facilitate the analysis.
- Problems are given to children in a regular classroom setting.

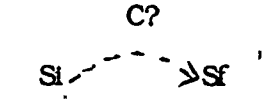
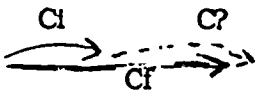
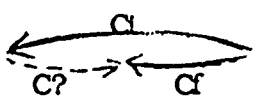
TASK

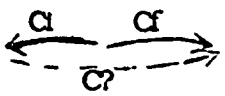
Protocol - At first, general information is given to the children in view of clarifying the conditions in which they will be placed and raising their interest to solve and show their solution procedures.

"We gave these problems to other children at the same grade level as you are, in another school. They found them difficult and, in fact, didn't perform very well. We are here to show you these problems and to see if you can solve them. We would especially like you to show how you manage to solve each one".

Booklets are distributed to each child with the instruction to solve the problems in the proposed sequence, and as each problem is solved to detach the page, place it upside-down on the corner of his desk where it will be picked up.

Examples of problems given to children to illustrate the increasing complexity in terms of the variable "problem structure" (cf. METHOD & SUBJECTS)

Structure	A typical problem	Problem given in grade:
 Unique change	1- I had 8 marbles in my pocket. A friend brought me some. Now I have got 17 marbles in my pocket. How many marbles has he brought me?	1rst 2nd
 Sequence of changes with no opposition	2- Michel is to step over stones in order to cross the river without getting is feet wet. From the bank he lands on a stone after jumping over 7 stones. He makes another jump to land on the opposite bank. How many stones does he jump over the second time?	1rst 2nd 3rd
 Sequence of changes with opposition. Ci and Cf in the same direction	3- Alain and his friends are playing a game in which they can win or lose counters. The game has been going on for a while and it's now Alain's turn again. He plays, loses 6 counters and is allowed an immediate other trial. Knowing that he now has 4 counters less than prior to these two last trials, tell me if Alain won or lost counters at his second trial and how many?	3rd 4th

 Sequence of changes with opposition Ci and Cf opposite legend: i initial f final C change	4- Alain and his friends are playing a game in which they can win or lose counters. The game has been going on for a while and it's now Alain's turn again. He plays, loses 7 counters and is allowed an immediate other trial. Knowing that he has now 3 counters more than prior to the last two trials, tell me if Alain won or lost counters at his second trial and how many?	4th 5th 6th

RESULTS

The percentages of success confirm that problems requiring the reconstruction of a transformation bring about troubles at the different levels of primary school (see table 1). For all problems of a given structure, the percentage of success increase with grade levels. However, analogous difficulties persist when the structure gets more complex.

Incorrect strategies used by young children to describe the change consist in either giving as answers the final or the initial states or in giving a combination of both. As we noted, they mainly focus on the states (see table 3). Similar strategies are observed with older children when they are faced with more complex structures in which a sequence of changes is involved since they handle the initial and the final changes as if they were states. As a consequence, in solving problems children do not distinguish structures in which changes are non opposite from the "unique change" structure. This is confirmed by the similarity of success rates for those type of structure (see the figure 1). For sequence of opposite changes, we haved noted a drop in the success rate but this drop is more dramatic when the resulting change is opposite to the initial change.

TABLE 1

Average % of successful answers for categories of problems determined by the variable "structure of the problem"

Grade Level / Problem structure	1st	2nd	3rd	4th	5th	6th
Unique change Si ? Sf	19,5%	45%	64%	84%	-----	-----
Sequence of changes with no opposition Ci → ? → Cf	24.5%	41%	-------	68.5%	-------	90%
Sequence of changes with opposition Ci ← ? → Cf Ci no Cf opposition	-------	-------	7%	27%	-------	49%
Sequence of changes with opposition Ci Cf ? Ci Cf opposite	-------	-------	-------	6.5%	13%	30.5%

FIGURE 1

% of successful answers for categories of problems

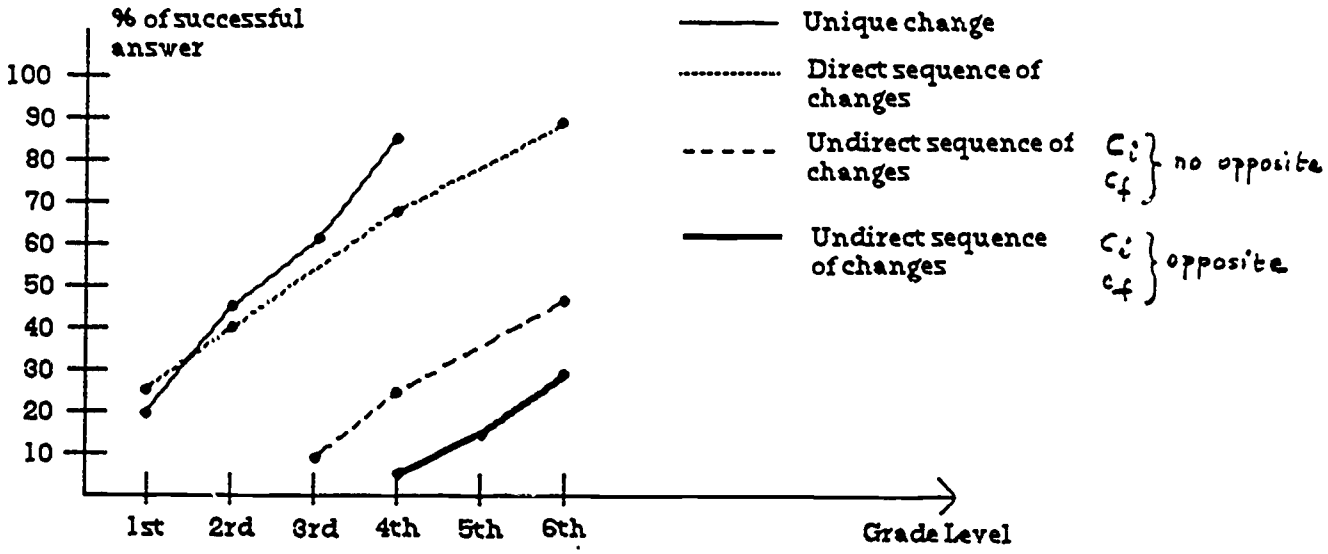


TABLE 2

% of successful answers to the four specific problems (ref. tasks)

Grade Level / Specific problems (ref. tasks)	1st	2nd	3rd	4th	5th	6th
No 1	21%	67%	64%	81.5%	-----	-----
No 2	24.5%	46%	-------	68.5%	-------	90%
No 3	-------	-------	7%	21%	-------	36%
No 4	-------	-------	-------	6.5%	12%	20.5%

TABLE 3

% of typical errors made in the four specific problems

	typical errors % of wrong answers in grade	Si + Sf	Sf	Need Si
No 1	33% in 2nd	10%	0%	Not relevent
No 2	64% in 2nd	15%	11,5%	0%
No 3	93% in 3rd 79% in 4th	3,5% 6%	18% 19%	0% 0%
No 4	93,5% in 4th 88% in 5th 79,5% in 6th	16% 25% 3,5%	55% 41% 31%	6,5% 0% 7%

CONCLUSION

Children have great difficulties with problems requiring the reconstruction of a transformation. For this reason and since they will face such problems at school in sciences and in other subject matters, we should pay more attention to that important issue.

The knowledge we now have of the incorrect strategies used by the children should help us in the design of interventions that would force them to concentrate on changes and make the states less "attractive". They should encourage children to be more attentive to a change unfolding in front of them since they are expected to reproduce this change from the initial state onwards. This is precisely what we mean by re-constructing a transformation.

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